PHYS 3102: Effective Field Theories in Particle Physics

Samuel Homiller
https://homiller-hep.net/teaching/2026-1/

Homework 1

Deadline: 02/13/2026

Problem 1: Reparameterization for a Simple Scalar EFT
Consider the following Lagrangian for a scalar field,

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\phi^4 + \frac{c_1}{\Lambda^2}\phi^3\Box\phi + \frac{c_2}{\Lambda^2}\phi^6\,. \tag{1}$$

1. Find a field redefinition $\phi \mapsto \phi'$ that yields a new Lagrangian $\mathcal{L}'$ in which the c_1 term is eliminated. You should find new λ' and c_2' terms; write these coefficients in terms of the original parameters.

2. Now, compute the equations of motion for ϕ from the original Lagrangian, $\mathcal{L}$. Show that, up to $\mathcal{O}(1/\Lambda^2)$, the same form for $\mathcal{L}'$ is obtained by applying the equations of motion to the c_1 term.

3. Compute the 4-point function for ϕ up to $\mathcal{O}(1/\Lambda^2)$ by evaluating the tree-level Feynman diagrams from both $\mathcal{L}$ and $\mathcal{L}'$ and show that the result is equivalent (using the relationships for λ' and c_2').

Problem 2: EFT of a Toy Scalar-Fermion Theory
Consider a model with a complex scalar field φ and Dirac fermion ψ, where both fields transform under a U(1) symmetry as:

$$\varphi \mapsto e^{i\alpha}\varphi\,, \qquad \psi \mapsto e^{i\alpha}\psi\,. \tag{2}$$

The renormalizable Lagrangian is given by

$$\mathcal{L} = \partial_\mu\varphi^\dagger\partial^\mu\varphi - m_\varphi^2\varphi^\dagger\varphi - \lambda(\varphi^\dagger\varphi)^2 + \bar{\psi}(i\gamma^\mu\partial_\mu - m_\psi)\psi\,. \tag{3}$$

This respects both U(1) and parity symmetry.

1. First, write down all the Hermitian operators of mass dimension 5 and 6, composed of ϕ, ψ and up to two derivatives. Ensure they respect Lorentz symmetry, the U(1) symmetry (2), and parity.

2. Now use the equations of motion, field redefinitions, Fierz transformations, etc. to identify a non-redundant set of operators up to $\mathcal{O}(1/\Lambda^2)$.

Problem 3: Redundant Terms in the SMEFT

As discussed in lecture, the dimension-six operator basis of Buchmüller and Wyler[1] included a number of redundant operators that can be eliminated using the SM equations of motion. Here we'll work out how several more of these operators can be eliminated.

1. First, work out the equations of motion for the gauge field $B_{\mu\nu}$ and the fermion fields L and $\bar{e}$ (use the SM in Weyl notation; don't forget about the Hermitian conjugate terms).

2. Now, consider the operator

$$\mathcal{O}_{De} = L^\dagger (D_\mu \bar{e})^\dagger D^\mu H \,. \tag{4}$$

   Use the equations of motion for the Higgs field and the Lepton fields derived above to rewrite this in terms of Warsaw basis operators in the ψ^4, $\psi^2 H^3$, $\psi^2 H^2 D$ and $\psi^2 X H$ class (along with shifts in the SM Yukawa terms and total derivatives).

3. Next consider the operator,

$$\mathcal{O}_{LB} = i L^\dagger \bar{\sigma}_\mu D_\nu L \, B^{\mu\nu} \,. \tag{5}$$

   Again show that this can be eliminated using the equations of motion for the fermion fields and the Bianchi identity, $D^\mu X_{\mu\nu} = 0$. You will also need the identity

$$\bar{\sigma}^\mu \sigma^\nu \bar{\sigma}^\rho = \eta^{\mu\nu} \bar{\sigma}^\rho - \eta^{\mu\rho} \bar{\sigma}^\nu + \eta^{\nu\rho} \bar{\sigma}^\mu - i \epsilon^{\mu\nu\rho\kappa} \bar{\sigma}_\kappa \,.$$

[1] W. Buchmuller and D. Wyler, "Effective Lagrangian Analysis of New Interactions and Flavor Conservation", Nucl. Phys. **B** 286, 621-653 (1986).